APPENDIX J

DERIVATION OF ALPHA SCANNING EQUATIONS
PRESENTED IN SECTION 6.7.2.2

For alpha survey instrumentation with a background around one to three counts per minute, a single count will give a surveyor sufficient cause to stop and investigate further. Assuming this to be true, the probability of detecting given levels of alpha emitting radionuclides can be calculated by use of Poisson summation statistics.

Discussion

Experiments yielding numerical values for a random variable X, where X represents the number of events occurring during a given time interval or a specified region in space, are often called Poisson experiments (Walpole and Myers 1985). The probability distribution of the Poisson random variable X, representing the number of events occurring in a given time interval t, is given by:

$$P(x;\lambda t) = \frac{e^{-\lambda t}(\lambda t)^{x}}{x!}, \quad x = 0, 1, 2,... \tag{J-1}$$

where:

$P(x; \lambda t)$ = probability of x events in time interval t
λ = Average number of events per unit time
λt = Average value expected

To define this distribution for an alpha scanning system, substitutions may be made giving:

$$P(n;m) = \frac{e^{-m}m^{n}}{n!} \tag{J-2}$$

where:

$P(n; m)$ = probability of getting n counts when the average number expected is m
m = λt , average number of counts expected
n = x, number of counts actually detected

For a given detector size, source activity, and scanning rate, the probability of getting *n* counts while passing over the source activity with the detector can be written as:

$$P(n;m) = \frac{e^{\frac{-GEd}{60v}}\left[\dfrac{GEd}{60v}\right]^n}{n!} = \frac{e^{\frac{-GEt}{60}}\left[\dfrac{GEt}{60}\right]^n}{n!} \qquad (J\text{-}3)$$

where:

G	=	source activity (dpm)
E	=	detector efficiency (4π)
d	=	width of the detector in the direction of scan (cm)
v	=	scan speed (cm/s)
t	=	d/v, dwell time over source (s)

If it is assumed that the detector background is equal to zero, then the probability of observing greater than or equal to 1 count, $P(n \geq 1)$, within a time interval t is:

$$P(n \geq 1) = 1 - P(n = 0) \qquad (J\text{-}4)$$

If it is also assumed that a single count is sufficient to cause a surveyor to stop and investigate further, then:

$$P(n \geq 1) = 1 - P(n = 0) = 1 - e^{-\frac{GEd}{60v}} \qquad (J\text{-}5)$$

Figures J.1 through J.3 show this function plotted for three different detector sizes and four different source activity levels. Note that the source activity levels are given in terms of areal activity values (dpm per 100 cm^2), the probe sizes are the dimensions of the probes in line with the direction of scanning, and the detection efficiency has been assumed to be 15%. The assumption is made that the areal activity is contained within a 100 cm^2 area and that the detector completely passes over the area either in one or multiple passes.

Once a count has been recorded and the surveyor stops, the surveyor should wait a sufficient period of time such that if the guideline level of contamination is present, the probability of getting another count is at least 90%. This minimum time interval can be calculated for given contamination guideline values by substituting the following parameters into Equation J-5 and solving:

$$P(\geq 1) \; = \; 0.9$$
$$d/v \; = \; t$$
$$G \; = \; \frac{CA}{100}$$

where:

C = contamination guideline (dpm/100 cm^2)
A = Detector area (cm^2)

Giving:

$$t \; = \; \frac{13800}{CAE} \tag{J-6}$$

Equation J-3 can be solved to give the probability of getting any number of counts while passing over the source area, although the solutions can become long and complex. Many portable proportional counters have background count rates on the order of 5 to 10 counts per minute and a single count will not give a surveyor cause to stop and investigate further. If a surveyor did stop for every count, and subsequently waited a sufficiently long period to make sure that the previous count either was or wasn't caused by an elevated contamination level, little or no progress would be made. For these types of instruments, the surveyor usually will need to get at least 2 counts while passing over the source area before stopping for further investigation. Assuming this to be a valid assumption, Equation J-3 can be solved for n ≥ 2 as follows:

$$P(n \geq 2) = 1 - P(n=0) - P(n=1)$$
$$= 1 - e^{-\frac{(GE+B)t}{60}} - \frac{(GE+B)t}{60} e^{-\frac{(GE+B)t}{60}} \tag{J-7}$$
$$= 1 - e^{-\frac{(GE+B)t}{60}} \left(1 + \frac{(GE+B)t}{60} \right)$$

Where:

P(n≥2) = probability of getting 2 or more counts during the time interval t
P(n=0) = probability of not getting any counts during the time interval t
P(n=1) = probability of getting 1 count during the time interval t
B = background count rate (cpm)

All other variables are the same as in Equation J-3.

Figures J-4 through J-7 show this function plotted for three different probe sizes and three different source activity levels. The same assumptions were made when calculating these curves as were made for Figures J-1 through J-3 except that the background was assumed to be 7 counts per minute.
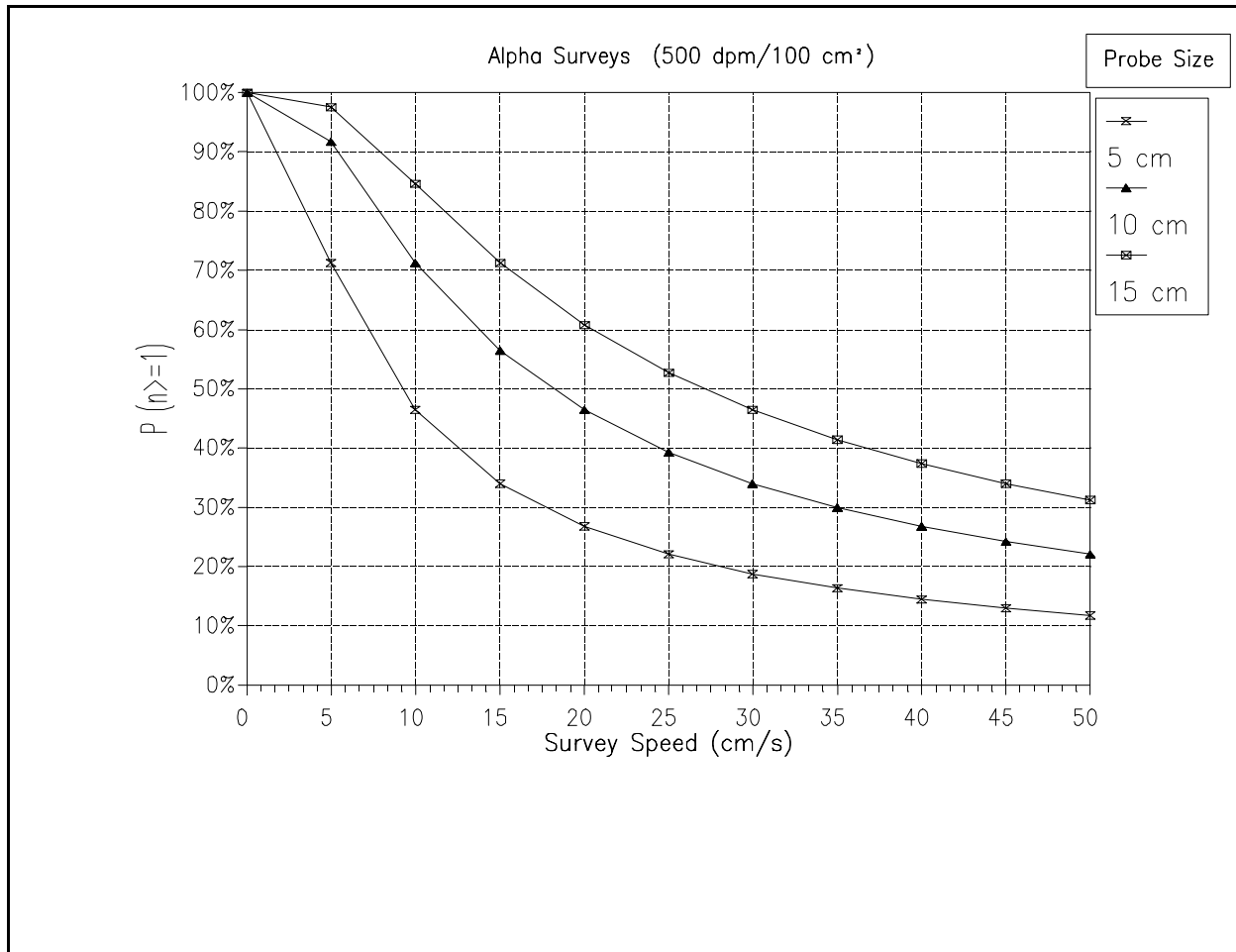
Figure J.1 **Probability (P) of getting one or more counts when passing over a 100 cm²
area contaminated at 500 dpm/100 cm² alpha. The chart shows the
probability versus scanning speed for three different probe sizes. The probe
size denotes the dimensions of the probes which are in line with the direction
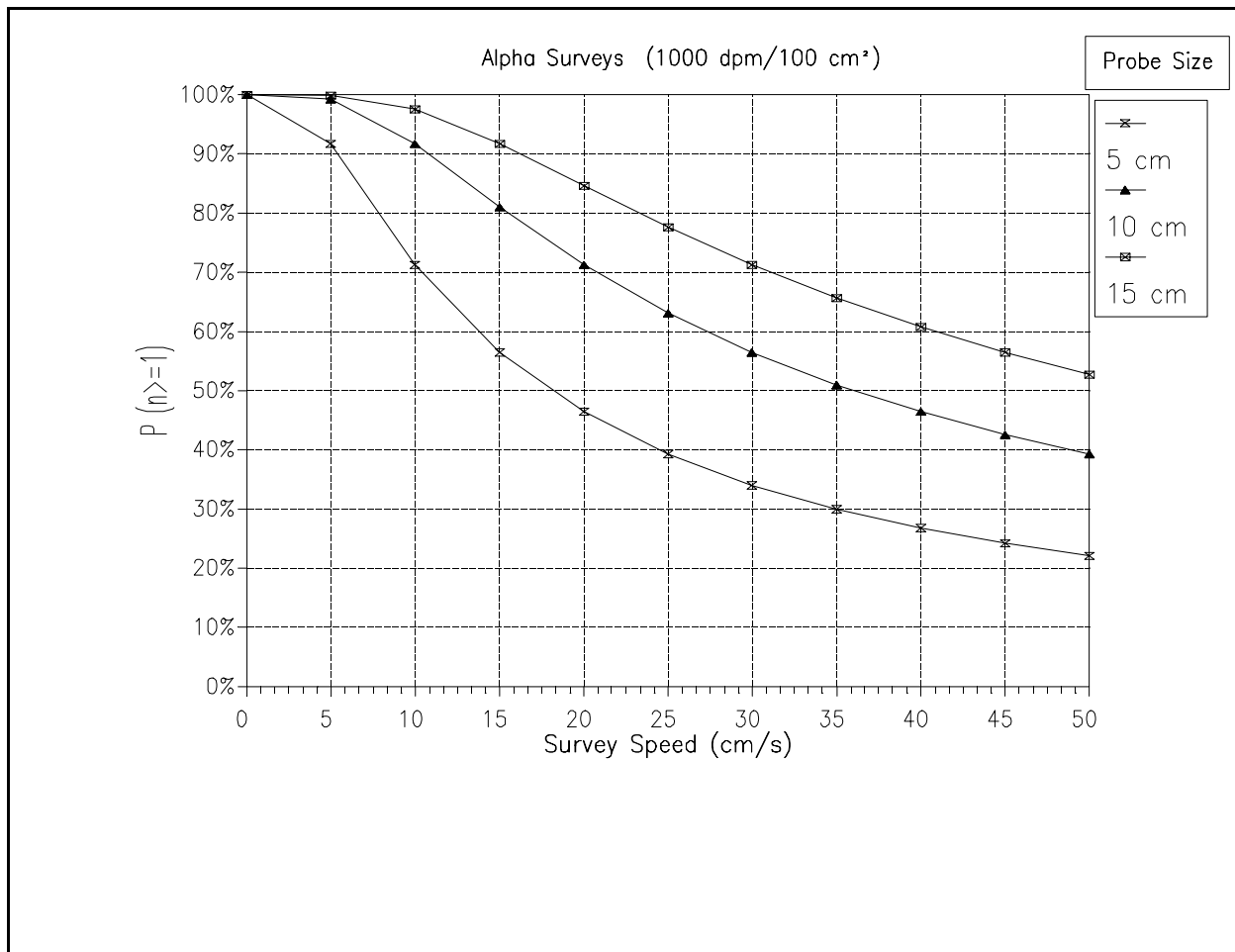of scanning. A detection efficiency of 15% (4π) is assumed.**

Figure J.2   **Probability (P) of getting one or more counts when passing over a 100 cm²
area contaminated at 1,000 dpm/100 cm² alpha. The chart shows the
probability versus scanning speed for three different probe sizes. The probe
size denotes the dimensions of the probes which are in line with the direction
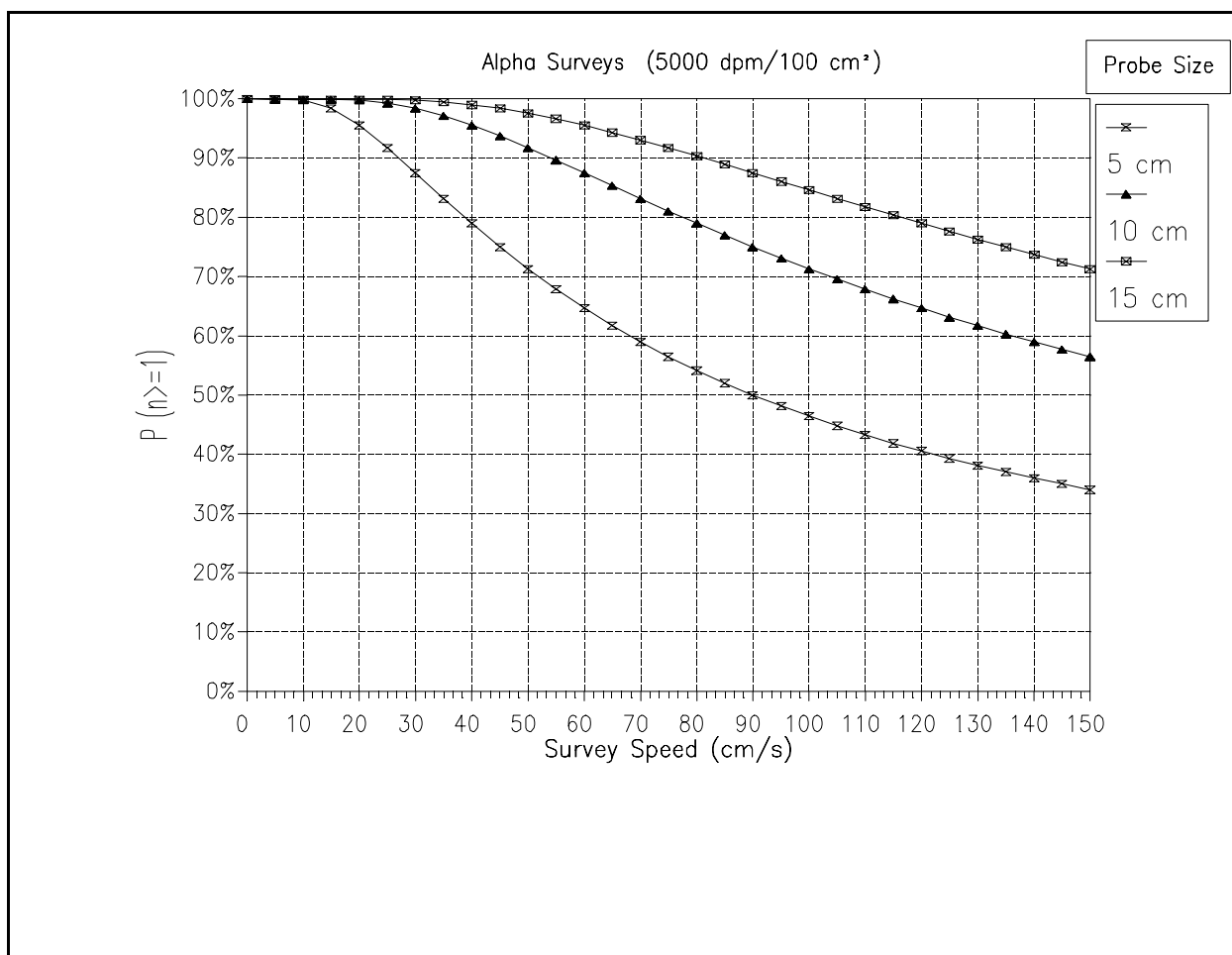of scanning. A detection efficiency of 15% (4π) is assumed.**

**Figure J.3   Probability (P) of getting one or more counts when passing over a 100 cm²
area contaminated at 5,000 dpm/100 cm² alpha. The chart shows the
probability versus scanning speed for three different probe sizes. The probe
size denotes the dimensions of the probes which are in line with the direction
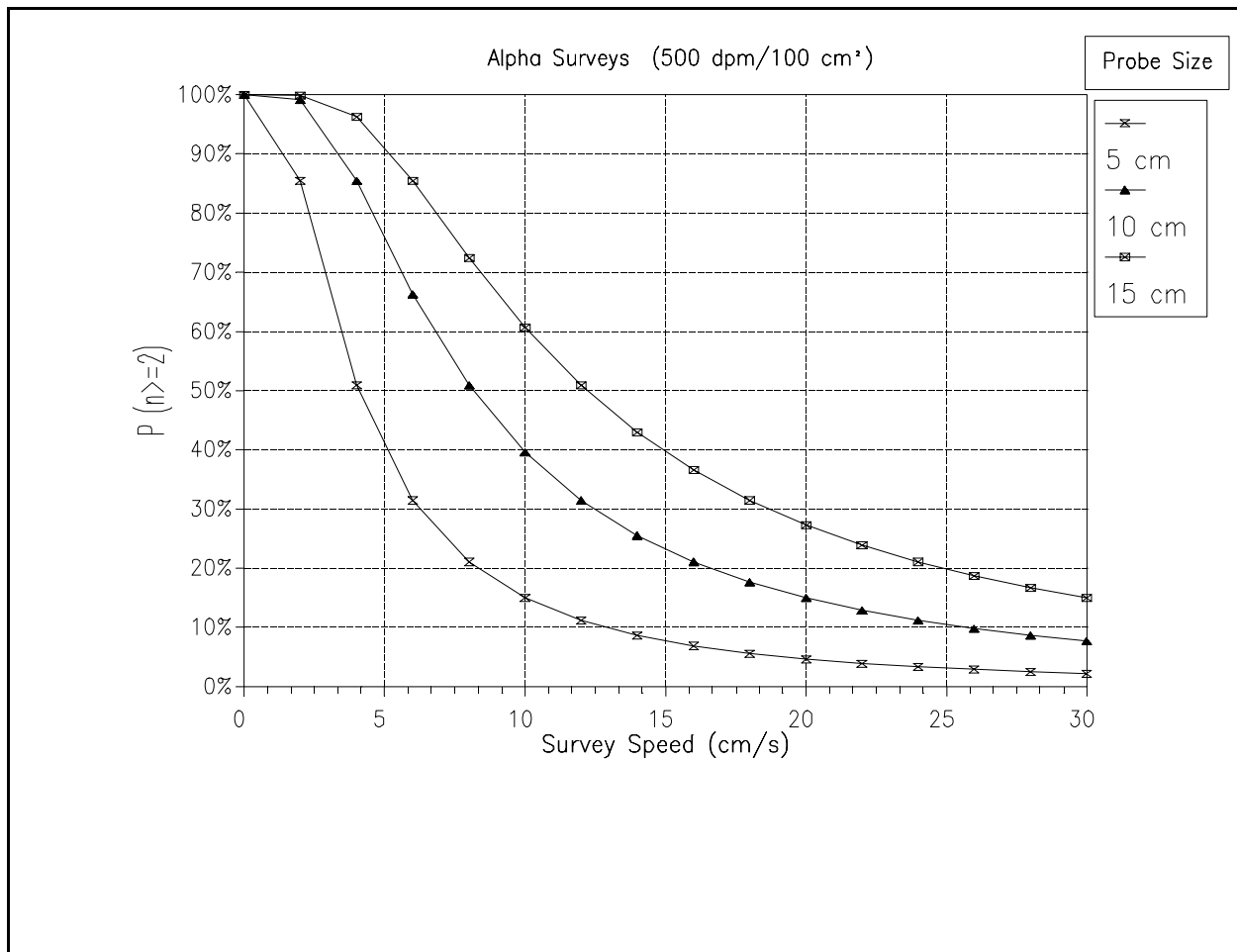of scanning. A detection efficiency of 15% (4π) is assumed.**

Figure J.4 **Probability (P) of getting two or more counts when passing over a 100 cm²
area contaminated at 500 dpm/100 cm² alpha. The chart shows the
probability versus scanning speed for three different probe sizes. The probe
size denotes the dimensions of the probes which are in line with the direction
of scanning. A detection efficiency of 15% (4π) is assumed.**
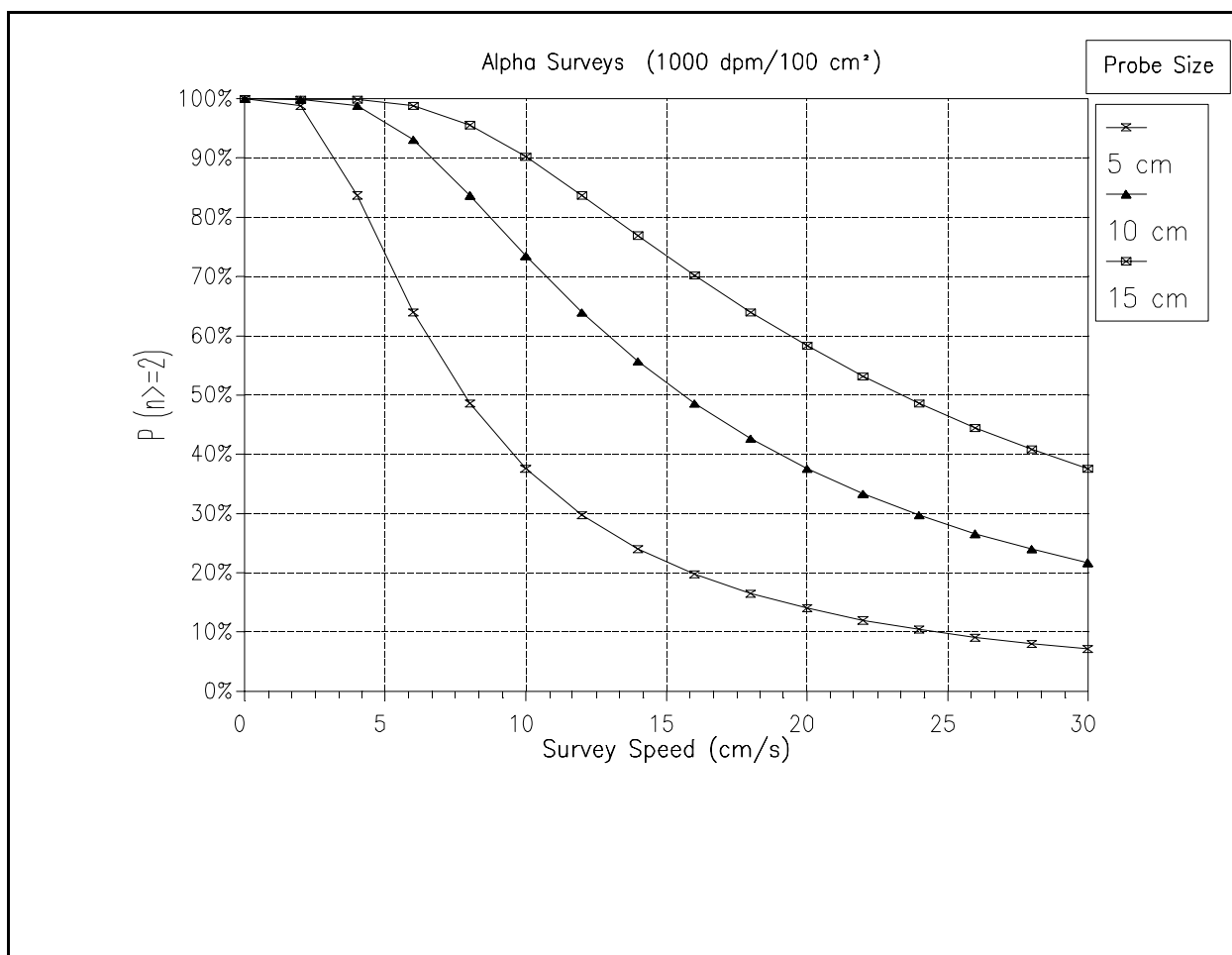
**Figure J.5   Probability (P) of getting two or more counts when passing over a 100 cm²
area contaminated at 1,000 dpm/100 cm² alpha. The chart shows the
probability versus scanning speed for three different probe sizes. The probe
size denotes the dimensions of the probes which are in line with the direction
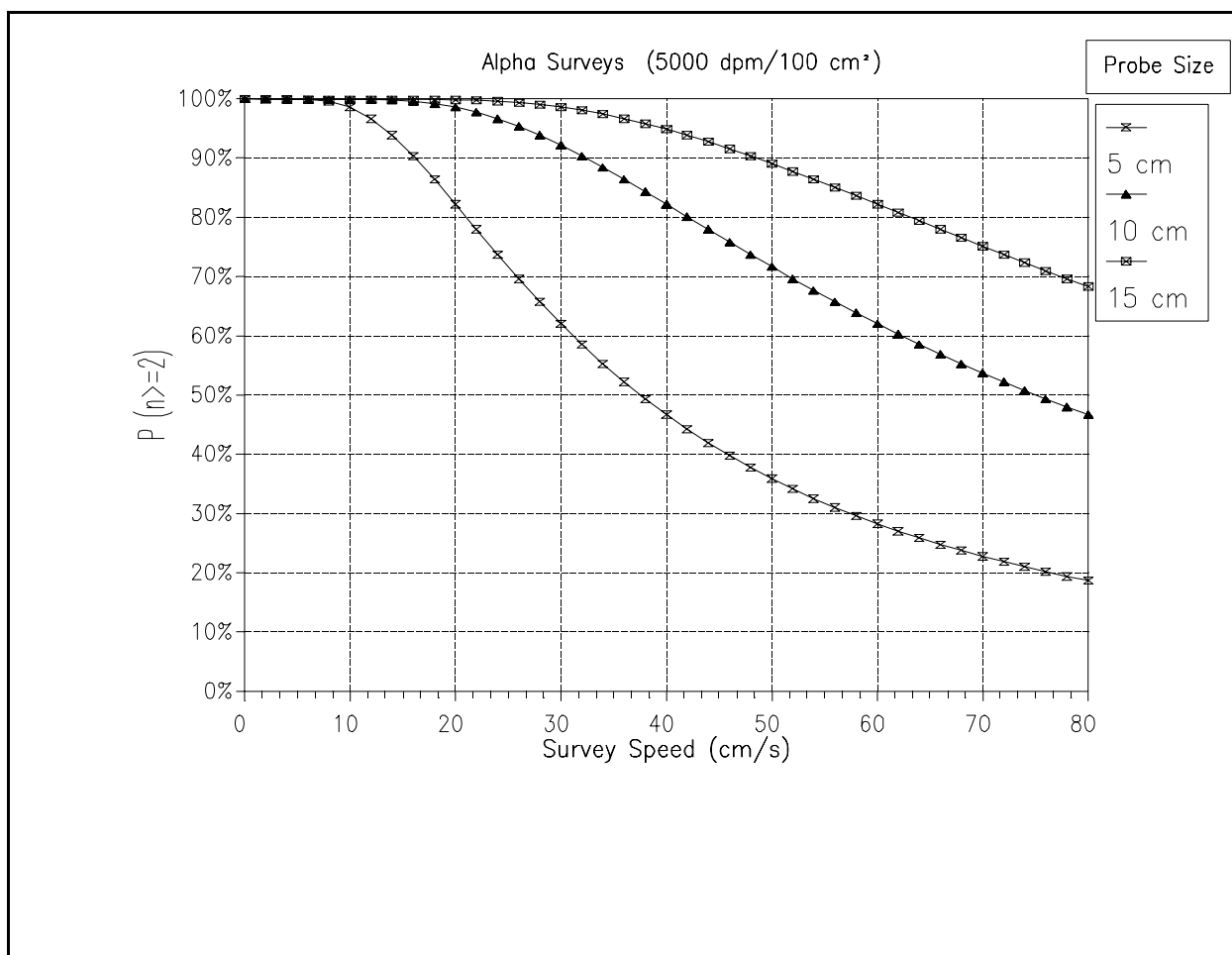of scanning. A detection efficiency of 15% (4π) is assumed.**

Figure J.6   **Probability (P) of getting two or more counts when passing over a 100 cm^2 area contaminated at 5,000 dpm/100 cm^2 alpha. The chart shows the probability versus scanning speed for three different probe sizes. The probe size denotes the dimensions of the probes which are in line with the direction of scanning. A detection efficiency of 15% (4π) is assumed.**